Lecture 10 - Oct. 8

Math Review, Bridge Controller Intro

Injective vs. Surjective vs. Bijective
Modelling an Array as a Function
?

Announcements/Reminders

- ProgTest1 tomorrow Wednesday
- Lab3 released

total fun ⇒ partial func.

Injective Functions

* it cannot be the case that two distinct domain values map to the same range value.

`isInjective(f)`

fun. prop.

$\forall s, t_1, t_2 \bullet (s \in S \land t_1 \in T \land t_2 \in T) \Rightarrow ((s, t_1) \in f \land (s, t_2) \in f \Rightarrow t_1 = t_2)$

$\forall s_1, s_2, t \bullet (s_1 \in S \land s_2 \in S \land t \in T) \Rightarrow$ * ** $\boxed{((s_1, t) \in f \land (s_2, t) \in f \Rightarrow s_1 = s_2)}$

inj. prop.

If f is a ***partial injection***, we write: $\boxed{f \in S \twoheadrightarrow\!\!\!\!\!\!\mapsto T}$

- e.g., $\{\varnothing, \{(1, \mathbf{a})\}, \{(2, \mathbf{a}), (3, \mathbf{b})\}\} \subseteq \{1, 2, 3\} \rightarrowtail\!\!\!\!\!\!+ \{a, b\}$
- e.g., $\underline{\{(1, \mathbf{b}), (2, a), (3, \mathbf{b})\}} \notin \{1, 2, 3\} \rightarrowtail\!\!\!\!\!\!+ \{a, b\}$ partial fun. not inj.
- e.g., $\{(1, \mathbf{b}), (3, \mathbf{b})\} \notin \{1, 2, 3\} \rightarrowtail\!\!\!\!\!\!+ \{a, b\}$ partial fun. not inj.

If f is a ***total injection***, we write: $\boxed{f \in S \rightarrowtail T}$

- e.g., $\{1, 2, 3\} \rightarrowtail \{a, b\} = \varnothing$
- e.g., $\{(2, d), (1, a), (3, c)\} \in \{1, 2, 3\} \rightarrowtail \{a, b, c, d\}$
- e.g., $\{(\mathbf{2}, d), (\mathbf{1}, c)\} \notin \{1, 2, 3\} \rightarrowtail \{a, b, c, d\}$
- e.g., $\{(2, \mathbf{d}), (1, c), (3, \mathbf{d})\} \notin \{1, 2, 3\} \rightarrowtail \{a, b, c, d\}$

$s_1 \quad t \quad s_2 \quad t$

e.g. $(\boxed{1}, \boxed{b}), (\boxed{3}, \boxed{b})$ witness of violating inj. prop.

true

$\boxed{(1, b) \in f \land (3, b) \in f} \Rightarrow \boxed{1 = 3}$ false

false

** Contrapositive: $\boxed{s_1 \neq s_2} \Rightarrow \neg((s_1, \boxed{t}) \in f \land (s_2, \boxed{t}) \in f)$ s_1 and s_2 cannot map to same ran. value.

distinct dom values

the set of all possible total injective functions

If f is a ***total injection***, we write: $f \in S \rightarrowtail T$

- e.g., $\{1,2,3\} \rightarrowtail \{a,b\} = \varnothing$ impossible to construct a total inj func from S and T
- e.g., $\{(2,d),(1,a),(3,c)\} \in \{1,2,3\} \rightarrowtail \{a,b,c,d\}$
- e.g., $\{(2,d),(1,c)\} \notin \{1,2,3\} \rightarrowtail \{a,b,c,d\}$
- e.g., $\{(2,d),(1,c),(3,d)\} \notin \{1,2,3\} \rightarrowtail \{a,b,c,d\}$

$\{1,2,3\} \rightarrowtail \{a,b\}$

construct injection

$\{(1,a),(2,b)\}$

inj. but not total

to make it total, we can add either $(3,a)$ or $(3,b)$, but it violates inj. prop.

① functional property ⇒ partial function.

② $\text{dom}(f) = S$

③ injective property.

	func. prop.	total	inj. prop.
①	✓	✓	✓
②	✓	X	✓
③	✓	✓	X

Surjective Functions

total $(f) \iff \text{dom}(f) = S$

$$\texttt{isSurjective}(f) \iff \underline{\text{ran}}(f) = \underline{T}$$

* func. prop. surjective prop.

** - func. prop - total - surjective

If f is a* **partial surjection**, we write: $\boxed{f \in S \twoheadrightarrow\!\!\!\!\!\!+\; T}$

- e.g., $\{ \{(1,\mathbf{b}),(2,\mathbf{a})\}, \{(1,\mathbf{b}),(2,\mathbf{a}),(3,\mathbf{b})\} \} \subseteq \{1,2,3\} \twoheadrightarrow\!\!\!\!\!\!+\; \{a,b\}$ total at the same time.
- e.g., $\{(2,\mathbf{a}),(1,\mathbf{a}),(3,\mathbf{a})\} \notin \{1,2,3\} \twoheadrightarrow\!\!\!\!\!\!+\; \underline{\{a,b\}}$ $\boxed{\text{partial fun}}$, $\underline{\text{not}}$ sur.
- e.g., $\{(2,\mathbf{b}),(1,\mathbf{b})\} \notin \{1,2,3\} \twoheadrightarrow\!\!\!\!\!\!+\; \boxed{\{a,b\}}$ partial fun, $\underline{\text{not}}$ sur.

If f is a** **total surjection**, we write: $\boxed{f \in S \twoheadrightarrow T}$

- e.g., ① $\{ \{(2,a),(1,b),(3,a)\},$ ② $\{(2,b),(1,a),(3,b)\} \} \subseteq \underline{\{1,2,3\}} \twoheadrightarrow \underline{\{a,b\}}$
- e.g., ③ $\{(2,a),(3,b)\} \notin \{1,\underline{2,3}\} \twoheadrightarrow \underline{\{a,b\}}$
- e.g., ④ $\{(2,\mathbf{a}),(3,\mathbf{a}),(1,\mathbf{a})\} \notin \underline{\{1,2,3\}} \twoheadrightarrow \underline{\{a,b\}}$

	fun. prop.	total	surj.
①	✓	✓	✓
②	✓	✓	✓
③	✓	✗	✓
④	✓	✓	✗

Bijective Functions

f is ***bijective***/***a bijection***/*one-to-one correspondence* if f is ***total***, ***injective***, and ***surjective***.

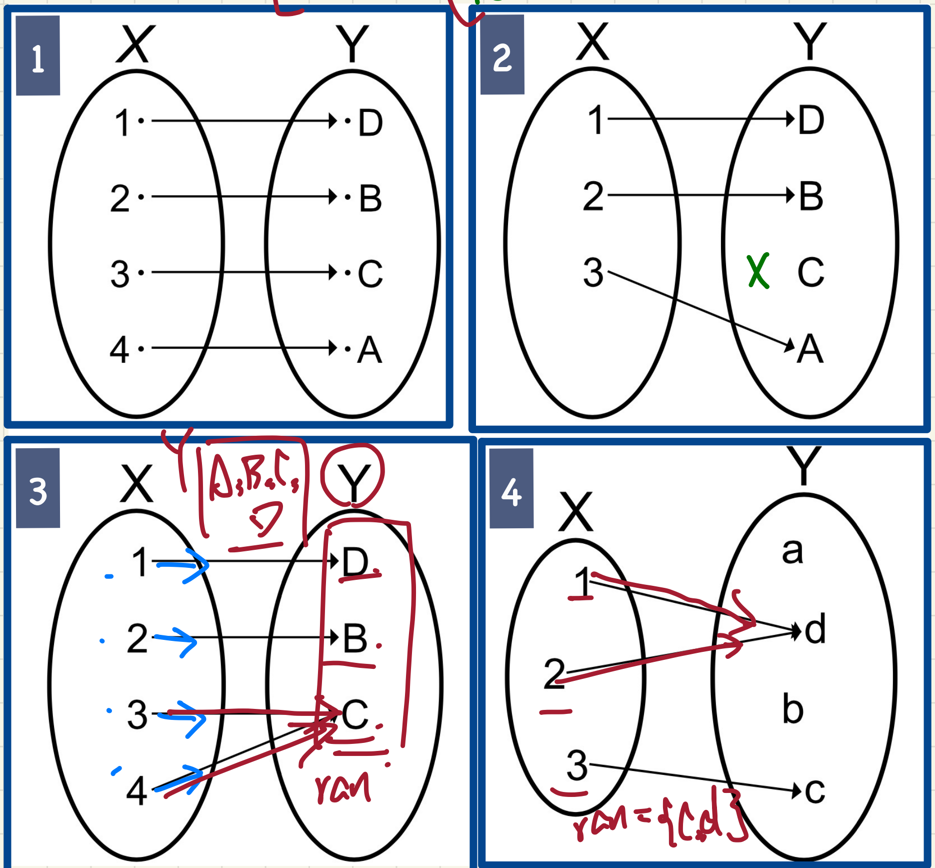
- e.g., $\{1,2,3\} \twoheadrightarrow \{a,b\} = \varnothing$ $\{(1,a),(2,b),(3,?)\}$
- e.g., $\{\ \{(1,a),(2,b),(3,c)\}, \{(2,a),(3,b),(1,c)\}\ \} \subseteq \{1,2,3\} \twoheadrightarrow \{a,b,c\}$
- e.g., ① $\{(\mathbf{2},b),(\mathbf{3},c),(\mathbf{4},a)\} \notin \{1,2,3,4\} \twoheadrightarrow \{a,b,c\}$
- e.g., ② $\{(1,\boxed{\mathbf{a}}),(2,b),(3,c),\boxed{(4,\mathbf{a})}\} \notin \{1,2,3,4\} \twoheadrightarrow \{a,b,c\}$
- e.g., ③ $\{(1,\boxed{\mathbf{a}}),(2,\boxed{\mathbf{c}})\} \notin \{1,2\} \twoheadrightarrow \{a,b,c\}$

	fun + total	inj	sur.
①	✗	✓	✓
②	✓	✗	✓
③	✓	✓	✗

Exercise

$Y = \{A, B, C, D\}$

$X = \{1, 2, 3, 4\}$



	1	2	3	4
partial	✓	✓	✓	✓
total	✓	✗	✓	✗
injection	✓	✓	✗	✗
surjection	✓	✗	✗	✗
bijection	✓	✗	✗	✗